

A Biography of Marvin Marcus

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Dedicated to Marvin Marcus

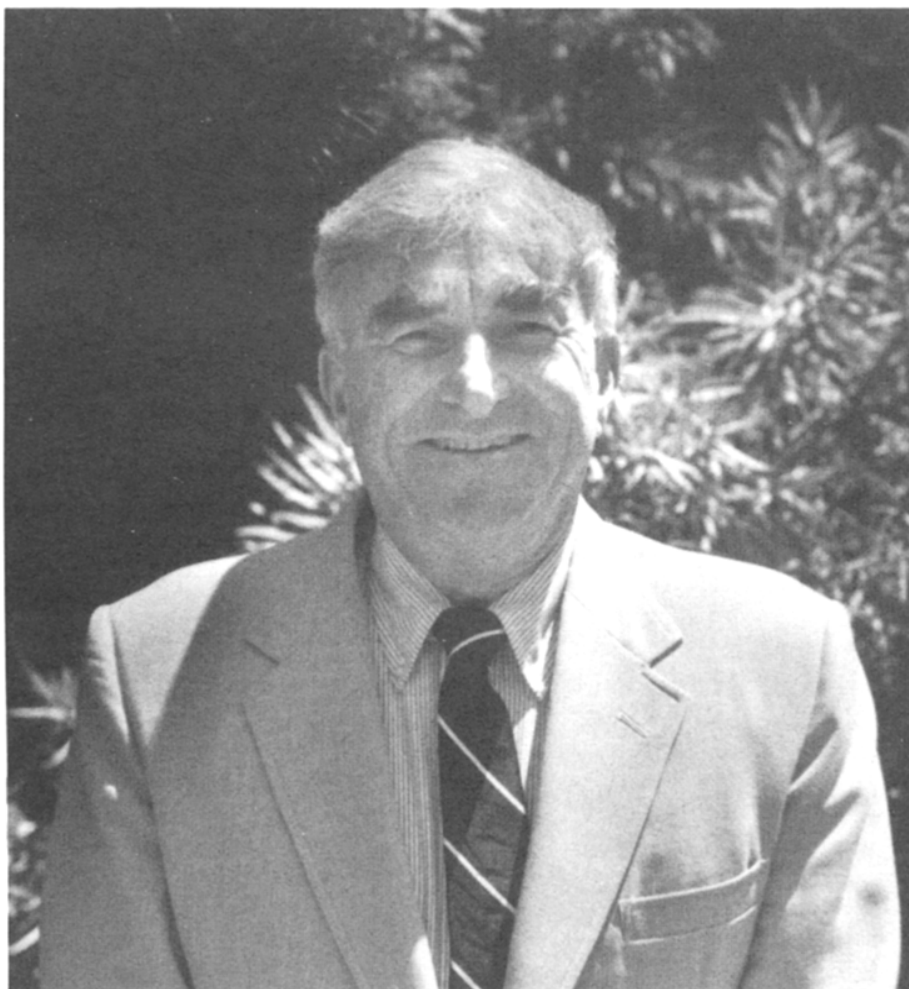
Submitted by Richard A. Brualdi

It came as something of a surprise to hear in 1991 of the retirement of Marvin Marcus from the University of California at Santa Barbara. For his students, colleagues, admirers, and even adversaries, it was difficult to imagine Marvin at half speed. Investigation revealed that Marvin was to continue teaching two classes at UCSB, as well as continue his research activities, tennis playing, book writing, and other pursuits. So, to paraphrase Mark Twain, while the reports of Marvin's retirement are highly exaggerated, it nonetheless is an appropriate time to honor our valued colleague who has contributed so much to the linear algebra community.

Marvin Marcus was born 31 July 1927 in Albuquerque, New Mexico, attended public schools in California, and served honorably in the United States Navy from 1944 to 1946. His mathematical career began at Berkeley, where he obtained a B.A. with highest honors in mathematics in 1950. Continuing at Berkeley, he obtained a Ph.D. in 1953 under the direction of Stephen Diliberto, with a specialization in the area of differential equations. As his publication list indicates, his interests turned quickly to linear algebra and matrix theory.

From 1954 to 1961 Marvin had positions at the University of British Columbia, and as a Research Mathematician at the National Bureau of Standards in Washington D.C. His research activity mushroomed during this period, and many lifelong friendships and professional relationships were established. He collaborated with many notables, including Phil Davis, Emilie Haynsworth, Henryk Minc, Morris Newman, and Olga Taussky. At the University of British Columbia he directed the master's thesis of R. C. Thompson, and supervised his first doctoral student, Roy Westwick.

In 1962, Marvin came, at last, to the University of California at Santa



Marvin Marcus

Barbara, where the major part of his career has been. His achievements there as researcher, teacher, author, and administrator have each been outstanding. Taken together, one wonders how a single person could have accomplished so much.

As an administrator, Marvin began by serving as chair of the mathematics department at UCSB from 1963 to 1968. This was a time of rapid growth at UCSB, during which Ky Fan, Henryk Minc, and R. C. Thompson joined the department among other fine mathematicians less well known in the linear algebra community. From 1973 to 1979, Marvin was Director of the Institute for Interdisciplinary Applications of Algebra and Combinatorics. From 1979 to 1986 he served as Coordinator of the Chinese Exchange Program, and was responsible for an influx of talented Chinese students at UCSB. From 1978 to 1986, Marvin served as Vice Chancellor and Dean of Research and Academic Development. The external funding to UCSB rose quickly and steadily with Marvin at the helm of research and development. In addition to administra-

tive duties for UCSB, a 1991 vita listed 44 grants which were obtained and administered by Marvin Marcus from 1966 to 1991, and which totaled in excess of \$3 million. These came from such diverse sources as IBM and NSF for purposes ranging from pure research to programs for secondary school teachers.

Marvin's authorship of scholarly texts has been prolific. A glance at the list of texts illustrates this point admirably. Some of these have been translated into Spanish and Russian. Three have been reprinted by Dover, including my favorite, *A Survey of Matrix Theory and Matrix Inequalities* (with Henryk Minc). The level of these texts ranges from precalculus to the definitive two volume *Finite Dimensional Multilinear Algebra*, and all are characterized by clarity and elegance. Taken alone, these texts would represent an admirable lifetime career achievement. The standards exhibited in Marvin's textbooks are also seen in his research and survey papers. In 1965 Marvin was awarded the MAA Editorial Prize for the article "Linear transformations on matrices," and in 1966 the L. R. Ford Memorial Prize for the article "Permanents."

Marvin has been an outstanding and energetic teacher, from the freshman to the Ph.D. level. He was unwilling to teach less than a full load of classes, even as department head or vice chancellor. He wrote the texts to be used in his undergraduate course, in which he was demanding, but highly respected by his students. His reputation among the graduate students made him sought after as a thesis advisor. Marvin's students were not known to drift or loaf. In spite of his incredible output of papers, books, grants, and so on, Marvin always had time to collar a student into his office, spend an hour or two giving direction and advice, and send the student off with more ideas (and work) than he or she knew what to do with, along with the admonition "Get cracking!" For those of Marvin's students not desirous of such attention, avoiding his office was no defense; Marvin would come to yours. Marvin made every effort to supply his graduate students with summer employment, such as working as a research assistant constructing a linear algebra bibliography, or teaching in one of the summer programs run by Marvin for the enrichment of high school teachers or of college mathematics teachers lacking a terminal degree. Marvin's students had a high success rate, and generally wrote nice theses. When graduation neared, Marvin would continue to assist and direct in that important last detail of a graduate education: finding a first job. Besides the 20 Ph.D. mathematicians who have completed their degrees under Marvin's supervision, there are a half dozen computer science students at UCSB who have studied under Marvin as well.

Until 1983 Marvin was in the Department of Mathematics at UCSB. From 1983 to 1987 he held joint appointments in the Departments of Mathematics and Computer Science, and since 1987 has been solely in the Department of Computer Science. Marvin's interest in computing at UCSB

dates to the early 1960s, when he served on university computing and statistics committees. (It is worth noting in passing that Marvin did not avoid service on university committees; the list of his committee service is substantial.) Marvin founded the Microcomputer Laboratory at UCSB in 1979 and served as Director until retirement. In the Department of Computer Science he has been active in curriculum development and the direction of graduate students. In 1989 he was given the Outstanding Computer Scientist Award at UCSB, which reflects the influence that Marvin has had upon computing there.

For those in the linear algebra community the research career of Marvin Marcus has left an indelible imprint on the field for which he will be long remembered. The accompanying publication list shows Marcus as a pioneer in several areas which enjoy intense current activity. The very first paper on the list concerns the numerical range, a topic to which Marvin has returned many times, and in which he has been an influential innovator. The permanent function is another area in which Marvin has had considerable impact. His work with Morris Newman stimulated two decades of activity on the van der Waerden conjecture concerning permanents of doubly stochastic matrices, which was finally resolved in 1980. His 1963 paper which established the permanental analog of the Hadamard determinant theorem asked a question which remains perhaps the most interesting open problem in pure linear algebra: the so-called *permanental dominance conjecture*, which has been worked on by Marvin's students as well as Elliot Lieb, Gordon James, Tom Pate, and many others. Marvin's prize-winning 1965 paper entitled "Permanents," in the *American Mathematical Monthly*, helped to popularize the permanent function within the mathematical community. Another prize-winning paper concerned linear transformations on matrices which preserve various invariants. Marvin here not only made many original contributions, but can be fairly credited with unifying many previous results and defining the area which today might be referred to as *linear preserver* problems. In multilinear algebra, Marvin's work has been a major force. He was a pioneer in using the techniques of multilinear algebra to obtain matrix inequalities. Geometry and inequalities are predominant themes in Marvin's work, and matrix inequalities represents yet another area in which his contributions have been profound. A quick look at the list of publications shows work on doubly stochastic matrices, Hadamard products, combinatorial matrix theory, and a host of other topics. In addition to this extremely influential body of work, Marvin has served on numerous editorial boards and has continually done more than his share of editing and refereeing. Particularly notable along these lines was the founding (with R. C. Thompson) of the journal *Linear and Multilinear Algebra*, which first appeared in March 1973.

Considering Marvin's prodigious output, one might suspect a harried overworked monomaniac to be behind it all. Such is not the case. Marvin has a lovely family with wife Becky and children Jeffrey and Karen, to whom he is a fond and devoted parent. He has a passion for tennis and was always loath to miss his daily court time while I was a graduate student. He has an excellent sense of humor, and is quick with a joke or quip. If you observe Marvin when entertaining at his home, a somewhat different personality emerges than that of Marvin at work. On campus, Marvin is an energetic blur, with seemingly no time to spare. At home, Marvin is a gracious and charming host, a sparkling conversationalist, and comes across as relaxed and at ease. My suspicion is that work resumes as soon as the last lingering guest is bid good night. For anyone in the linear algebra community, Marvin Marcus is an important and influential figure. For those of us fortunate enough to have worked with Marvin, he will always remain an inspiration not only for his work, but for the way in which he has done it. We wish him a long and productive retirement.

GRADUATE STUDENTS OF MARVIN MARCUS

The following mathematicians have completed their Ph.D. work under the direction of Marvin Marcus.

Professor Roy Westwick
University of British Columbia
Vancouver, B.C., Canada
Thesis: Linear transformations of Grassmann algebras
1960

Professor Nisar A. Khan
Muslim University
Aligarh, India
Thesis: Matrix commutators
1961

Associate Professor Peter Botta
University of Toronto
Toronto, Ontario, Canada
Thesis: Linear transformations on algebras
1965

Professor Stanley G. Williamson
University of California
San Diego, California
Thesis: Tensor algebras
1965

Professor William R. Gordon
Department of Mathematics
University of Victoria
Victoria, B.C., Canada
Thesis: Inequalities for generalized matrix functions
1965

Dr. George Soules
Institute for Defense Analyses
Princeton, New Jersey
Thesis: Combinatorial functions
1966

Dr. Paul J. Nikolai, Mathematician
Wright-Patterson Air Force Base
Thesis: Mean value properties of generalized matrix functions
[This thesis was supervised jointly with Professor H. J. Ryser (deceased), California
Institute of Technology]
1966

Professor Stephen J. Pierce
San Diego State University
San Diego, California
Thesis: Generalized isometries
1968

Professor William Watkins
California State University, Northridge
Northridge, California
Thesis: Inequalities for derivation operators on a tensor space
1969

Professor Russell Merris
California State University at Hayward
Hayward, California
Thesis: A generalization of the associated transformation
1969

Associate Professor Mohammad Shafat Ali
California State University at Long Beach
Long Beach, California
Thesis: Additive commutators, Jordan products and bilinear functions
1970

Dr. Elizabeth Wilson, Mathematician
Naval Laboratories
Point Mugu, California
Thesis: Partial derivations on symmetry classes of tensors
1971

Professor James Holmes
Bethel College
St. Paul, Minnesota
Thesis: Application of derivations to invariance problems
1971

Dr. Herbert Robinson
Independent Computer Consultant
Oceanside, California
Thesis: Quadratic & bilinear forms on symmetry classes of tensors
1975

Associate Professor Patricia Andresen
University of Alaska
Fairbanks, Alaska
Thesis: The finite dimensional numerical range
1976

Professor Robert Grone
San Diego State University
San Diego, California
Thesis: Isometries of matrix algebras
1976

Dr. Ivan Filippenko, Research Mathematician
The Aerospace Corporation
Los Angeles, California
Thesis: Higher and decomposable numerical ranges
1977

Professor John Choliet
Towson State University
Towson, Maryland
Thesis: Equalities of decomposable symmetrized tensors
1979

Dr. Kenneth Moore
Radar Systems Group
Hughes Aircraft Company
El Segundo, California
Thesis: Determinantal inequalities
1980

Dr. Kent Kidman
Hughes Aircraft Company
El Segundo, California
Thesis: Stochastic matrices and unitarily invariant norms
1983

BOOKS OF MARVIN MARCUS

- 1 *Lectures on Tensor and Grassman Products*, Lecture notes, Univ. of British Columbia, 1959.
- 2 *Basic Theorems in Matrix Theory*, Appl. Math. Ser. 57, National Bureau of Standards, 1959.
- 3 *Introduction to Linear Algebra* (with H. Minc), Macmillan, New York, 1965.
- 4 *Modern University Algebra* (with H. Minc), Macmillan, New York, 1966.
- 5 *New College Algebra* (with H. Minc), Houghton Mifflin, Boston, 1968.
- 6 *Elementary Linear Algebra* (with H. Minc), Macmillan, New York, 1968.
- 7 *Elementary Functions and Coordinate Geometry* (with H. Minc), Houghton Mifflin, Boston, 1969.
- 8 *A Survey of Finite Mathematics*, Houghton Mifflin, Boston, 1969.
- 9 *Algebra and Trigonometry* (with H. Minc), Houghton Mifflin, Boston, 1970.
- 10 *College Algebra* (with H. Minc), Houghton Mifflin, Boston, 1970.
- 11 *A Survey of Matrix Theory and Matrix Inequalities* (with H. Minc), Allyn & Bacon, 1964; paperback ed., Complementary Ser. in Math. 14, Prindle, Weber & Schmidt, 1970.
- 12 *College Trigonometry* (with H. Minc), Houghton Mifflin, Boston, 1971.
- 13 *Integrated Analytic Geometry and Algebra with Circular Functions* (with H. Minc), General Learning Corp., 1973.
- 14 *Finite Dimensional Multilinear Algebra, Part I*, Marcel Dekker, New York, 1973.
- 15 *Algebra*, Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1974.
- 16 Editor, *Proceedings of the 1974 National Science Foundation Conference on Theoretical Matrix Theory*, Algebra Inst., Univ. of California, Santa Barbara, 1974.
- 17 *Linear Algebra*, Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1974.
- 18 *Finite Dimensional Multilinear Algebra, Part II*, Marcel Dekker, New York, 1975.
- 19 *Structural Balance in Group Networks* (with C. Hubell and E. C. Johnsen), Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1975.
- 20 *Notes on Elementary Linear Algebra*, Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1976.
- 21 *Elementary Linear Algebra*, Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1976.
- 22 *Introduction to Modern Algebra*, Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1977.
- 23 *Topics in Linear Algebra*, Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1977.
- 24 *Introduction to Modern Algebra*, Marcel Dekker, New York, 1978.
- 25 *Notes on Elementary Linear Algebra*, calculator ed., Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1978.
- 26 *College Trigonometry* (with H. Minc), Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1979.

- 27 *Modern University Algebra: A Computational Approach Using BASIC*, Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1980.
- 28 *College Algebra: A Computational Approach Using BASIC*, Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1980.
- 29 *Elementary Linear Algebra: A Computational Approach Using BASIC*, Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1980.
- 30 *Elementary Functions and Trigonometry: A Computational Approach Using BASIC*, Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1980.
- 31 *Decisional Analysis: A Computational Approach Using BASIC*, Algebra Inst. Monograph Ser., Univ. of California, Santa Barbara, 1981.
- 32 *Computing without Mathematics: Basic, Pascal Applications* (with J. Marcus), Microcomputer Educational Materials, Santa Barbara, Calif., 1982.
- 33 *Discrete Mathematics: A Computational Approach Using Basic*, Computer Science Press, Rockville, Md., 1986.
- 34 *MacAlgebra, Basic Algebra on the MacIntosh*, Computer Science Press, Rockville, Md., 1986.
- 35 *An Introduction to Pascal and Precalculus*, Computer Science Press, Rockville, Md., 1986.
- 36 *Computing without Mathematics: Basic, Pascal Applications* (with J. Marcus), Computer Science Press, Rockville, Md., 1986.
- 37 *A Brief Survey of Numerical Computation Microcomputer Materials*, Santa Barbara, Calif., 1986.
- 38 *Introduction to Linear Algebra* (with H. Minc), Dover, New York, 1988.
- 39 *Foundations of Numerical Linear Algebra I*, Monographs Sci. Comput., Center for Computational Sciences and Engineering, Univ. of California, Santa Barbara, 1988.
- 40 *Scientific Computing with THINK's Lightspeed Pascal*, Microcomputer Materials, 1990.
- 41 *Computing without Math*, 2nd ed., Microcomputer Materials, 1990.
- 42 *Introduction to MatLabTM, A Primer*, Microcomputer Materials, 1990.
- 43 *Topics in Linear Algebra Using MatLabTM*, Microcomputer Materials, 1990.
- 44 *Matrices and MatlabTM: A Tutorial*, Microcomputer Materials, 1991.
- 45 *Matrices and MatlabTM: A Tutorial*, Prentice-Hall, 1993.
- 46 *A Survey of Matrix Theory & Matrix Inequalities*, Dover, 1992.
- 47 *A Survey of Finite Mathematics*, Dover, 1993.

PUBLICATIONS OF MARVIN MARCUS

- 1 Field convexity of a square matrix (with B. N. Moys), *Proc. Amer. Math. Soc.* 6:981–983 (1955).
- 2 A remark on a norm inequality for square matrices, *Proc. Amer. Math. Soc.* 6:117–119 (1955).
- 3 Some results on the asymptotic behavior of linear systems, *Canad. J. Math.* 7:531–538 (1955).

- 4 Boundedness of a continuous function, *Amer. Math. Monthly* 62 (1955).
- 5 An invariant surface theorem for a non-degenerate system. Contributions to non-linear oscillations, *Ann. of Math. Stud.* 36:243–256 (1956).
- 6 A note on the existence of periodic solutions of differential equations (with S. P. Diliberto), *Ann. of Math. Stud.* 36:237–241 (1956).
- 7 Repeating solutions for a degenerate system, *Ann. of Math. Stud.* 36:261–268 (1956).
- 8 On the optimum gradient method for systems of linear equations, *Proc. Amer. Math. Soc.* 1:77–81 (1956).
- 9 Extremal properties of Hermitian matrices (with J. McGregor), *Canad. J. Math.* 8:524–531 (1956).
- 10 An eigenvalue inequality for the product of normal matrices, *Amer. Math. Monthly* 63:173–174 (1956).
- 11 On the maximum principle of Ky Fan (with B. Moyls), *Canad. J. Math.* 9:313–320 (1957).
- 12 Inequalities for symmetric functions and Hermitian matrices (with L. Lopes), *Canad. J. Math.* 9:304–312.
- 13 A note on symmetric functions of eigenvalues (with R. C. Thompson), *Duke Math. J.* 24:97–99 (1957).
- 14 A note on the values of a quadratic form, *J. Wash. Acad. Sci.* 47:97–99 (1957).
- 15 Some extreme value results for indefinite Hermitian matrices I (with B. Moyls and R. Westwick), *Illinois J. Math.* 1:449–457 (1957).
- 16 On subdeterminants of doubly stochastic matrices, *Illinois J. Math.* 1:583–590 (1957).
- 17 A determinantal inequality of H. P. Robertson II, *J. Wash. Acad. Sci.* 47:264–266 (1957).
- 18 Maximum and minimum values for the elementary symmetric functions of Hermitian forms (with B. Moyls), *J. London Math. Soc.* 32:375–377 (1957).
- 19 Convex functions of quadratic forms, *Duke J. Math.* 24:321–326 (1957).
- 20 Some extreme value results for indefinite Hermitian matrices II (with B. Moyls and R. Westwick), *Illinois J. Math.* 2:408–414 (1958).
- 21 On a determinantal inequality, *Amer. Math. Monthly* 65:266–268 (1958).
- 22 On doubly stochastic transforms of a vector, *Quart. J. Math. Oxford* 2:74–80 (1958).
- 23 On the minimum of the permanent of a doubly stochastic matrix (with M. Newman), *Duke J. Math.* 26:61–72 (1959).
- 24 Convexity of the field of a linear transformation (with A. Goldman), *Canad. Math. Bull.* 2:15–18 (1959).

- 25 Linear transformations on algebras of matrices (with B. Moys), *Canad. J. Math.* 11:61–66 (1959).
- 26 All linear operators leaving the unitary group invariant, *Duke J. Math.* 26:155–163 (1959).
- 27 Extremal properties of Hermitian matrices II (with B. Moys and R. Westwick), *Canad. J. Math.* 11:379–382 (1959).
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- 30 Transformations on tensor product spaces (with B. Moys), *Pacific J. Math.* 9:1215–1221 (1959).
- 31 Diagonals of doubly stochastic matrices (with R. Ree), *Quart. J. Math. Oxford* 10:296–302 (1959).
- 32 On matrix commutators (with N. Khan), *Canad. J. Math.* 12:269–277 (1960).
- 33 Space of k -commutative matrices (with N. Khan), *J. Res. Nat. Bur. Standards* 64B:51–54 (1960).
- 34 Some properties and applications of doubly stochastic matrices, *Amer. Math. Monthly* 67:215–220 (1960).
- 35 A note on a group defined by a quadratic form (with N. Khan), *Canad. Math. Bull.* 3:143–148 (1960).
- 36 Linear maps on skew-symmetric matrices; the invariance of elementary symmetric functions (with R. Westwick), *Pacific J. Math.* 10:917–924 (1960).
- 37 The maximum number of equal non-zero subdeterminants (with H. Minc), *Arch. Math. (Basel)* 11:95–100 (1960).
- 38 Permanents of doubly stochastic matrices (with M. Newman), in *Proc. Sympos. Appl. Math.* 10, Amer. Math. Soc., 1960.
- 39 On a commutator result of Taussky and Zassenhaus (with N. Khan), *Pacific J. Math.* 10:1337–1346 (1960).
- 40 On a theorem of I. Schur concerning matrix transformations (with F. May), *Arch. Math.* 11:401–404 (1960).
- 41 On the unitary completion of a matrix (with P. Greiner), *Illinois J. Math.* 5:152–158 (1961).
- 42 Some generalizations of Kantorovich's inequality (with N. Khan), *Portugal. Math.* 20:33–38 (1961).
- 43 Another extension of Heinz's inequality, *J. Res. Nat. Bur. Standards* 65B:129–130 (1961).
- 44 A note on normal matrices (with N. Khan), *Canad. Math. Bull.* 4:23–27 (1961).

- 45 Symmetric means and matrix inequalities (with P. Bullen), *Proc. Amer. Math. Soc.* 12:285–290 (1961).
- 46 Bound for the p -condition number of matrices with positive roots (with P. Davis and E. Haynsworth), *J. Res. Nat. Bur. Standards* 65:13–14 (1961).
- 47 The permanent function as an inner product (with M. Newman), *Bull. Amer. Math. Soc.* 67:223–224 (1961).
- 48 Comparison theorems for symmetric functions of characteristic roots, *J. Res. Nat. Bur. Standards* 65:113–116 (1961).
- 49 Some results on non-negative matrices (with H. Minc and B. Moyls), *J. Res. Nat. Bur. Standards* 65:205–209 (1961).
- 50 On the relation between the determinant and the permanent (with H. Minc), *Illinois J. Math.* 5:376–381 (1961).
- 51 The sum of the elements of the powers of a matrix (with M. Newman), *Pacific J. Math.* 12:627–635 (1962).
- 52 Some results on doubly stochastic matrices (with H. Minc), *Proc. Amer. Math. Soc.* 13:571–579 (1962).
- 53 An inequality connecting the p -condition number and the determinant, *Numer. Math.* 4:350–353 (1962).
- 54 Linear operations on matrices, *Amer. Math. Monthly* 69:837–847 (1962).
- 55 The maximum number of zeros on the powers of an indecomposable matrix (with F. May), *Duke Math J.* 29:581–588 (1962).
- 56 The permanent function (with F. May), *Canad. J. Math.* 14:177–189 (1962).
- 57 Permanent preservers on the space of doubly stochastic matrices (with B. Moyls and H. Minc), *Canad. J. Math.* 14:190–194 (1962).
- 58 Matrices in linear mechanical systems, *Canad. Math. Bull.* 5:253–257 (1962).
- 59 The invariance of symmetric functions of singular values (with H. Minc), *Pacific J. Math.* 12:327–332 (1962).
- 60 The pythagorean theorem in certain symmetry classes of tensors (with H. Minc), *Trans. Amer. Math. Soc.* 104:510–515 (1962).
- 61 Hermitian forms and eigenvalues, in *Survey of Numerical Analysis* (J. Todd, Ed.), McGraw-Hill, 1962, pp. 198–313.
- 62 Inequalities for the permanent function (with M. Newman), *Ann. of Math.* 76:47–62 (1962).
- 63 Disjoint pairs of sets and incidence matrices (with H. Minc), *Illinois J. Math.* 7:137–147 (1963).
- 64 Another remark on a result of K. Goldberg, *Canad. Math. Bull.* 6:7–9 (1963).

- 65 Equality in certain inequalities (with A. Cayford), *Pacific J. Math.* 2:1319–1329 (1963).
- 66 The field of values of the Hadamard product (with R. C. Thompson), *Archiv. Math. (Basel)* 14:283–288 (1963).
- 67 Solution to advanced problem 5005: Rank of a matrix, *Amer. Math. Monthly* 70:337 (1963).
- 68 The permanent analogue of the Hadamard determinant theorem, *Bull. Amer. Math. Soc.* 69:494–496 (1963).
- 69 Generalizations of some combinatorial inequalities of H. J. Ryser (with W. R. Gordon), *Illinois J. Math.* 7:582–592 (1963).
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- 71 The Hadamard theorem for permanents, *Proc. Amer. Math. Soc.* 15:967–973 (1964).
- 72 The minimal polynomial of a commutator, *Portugal. Math.* 25:73–76 (1964).
- 73 Compounds of skew-symmetric matrices (with A. Yaqub), *Canad. J. Math.* 16:473–478 (1964).
- 74 Inequalities for subpermanents (with W. R. Gordon), *Illinois J. Math.* 8:607–614 (1964).
- 75 The use of multilinear algebra for proving matrix inequalities, in *Proceedings of the Conference on Matrix Theory*, Univ. of Wisconsin Press, Madison, 1964.
- 76 Inequalities for mappings on spaces of skew-symmetric tensors (with W. R. Gordon), *Duke Math. J.* 31:691–696 (1964).
- 77 Inequalities for general matrix functions (with H. Minc), *Bull. Amer. Math. Soc.* 70:308–313 (1964).
- 78 On two classical results of I. Schur, *Bull. Amer. Math. Soc.* 70:685–688 (1964).
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- 80 Diagonal products in doubly stochastic matrices (with H. Minc), *Quart. J. Math. Oxford* 16:32–34 (1965).
- 81 Generalized functions of symmetric matrices (with M. Newman), *Proc. Amer. Math. Soc.* 16:826–830 (1965).
- 82 Matrix applications of a quadratic identity for decomposable symmetrized tensors, *Bull. Amer. Math. Soc.* 71:360–364 (1965).
- 83 A sub-determinant inequality (with H. Minc), *Pacific J. Math.* 15:921–924 (1965).
- 84 Harnack's and Weyl's inequalities, *Proc. Amer. Math. Soc.* 16:864–866 (1965).

- 85 Generalized matrix functions (with H. Minc), *Trans. Amer. Math. Soc.* 116:316–329 (1965).
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- 87 The Cauchy-Schwarz inequality in the exterior algebra, *Quart. J. Math. Oxford* 17:61–63 (1966).
- 88 A permanental inequality—the case of equality (with H. Minc), *Canad. J. Math.* 18:1085–1090 (1966).
- 89 An inequality for the elementary symmetric functions of characteristic roots (with H. Minc), *Proc. Amer. Math. Soc.* 17:510–514 (1966).
- 90 On a classical commutator result (with R. C. Thompson), *J. Math. and Mech.* 16:583–588 (1966).
- 91 Permutations on symmetry classes (with H. Minc), *J. Algebra* 15:59–71 (1967).
- 92 Lengths of tensors, in *Inequalities*, Academic, New York, 1967, pp. 163–176.
- 93 Doubly stochastic associated matrices (with M. Newman), *Duke J. Math.* 34:591–597 (1967).
- 94 Some inequalities for combinatorial matrix functions (with G. W. Soules), *J. Combin. Theory* 2:145–163 (1967).
- 95 An inequality for linear transformations, *Proc. Amer. Math. Soc.* 18:793–797 (1967).
- 96 On a conjecture of B. L. Van der Waerden (with H. Minc), *Proc. Cambridge Philos. Soc.* 63:305–309 (1967).
- 97 A theorem on rank with applications to mappings on symmetry classes of tensors, *Bull. Amer. Math. Soc.* 73:675–677 (1967).
- 98 On a combinatorial result of R. A. Brualdi and M. Newman (with S. Pierce), *Canad. J. Math.* 20:1056–1067 (1968).
- 99 Extensions of the Minkowski inequality (with S. Pierce), *Linear Algebra Appl.* 1:13–27 (1968).
- 100 Extensions of classical matrix inequalities (with H. Minc), *Linear Algebra Appl.* 1:421–444 (1968).
- 101 Elementary divisors of associated transformations (with S. Pierce), *Linear Algebra Appl.* 2:21–35 (1969).
- 102 Matrices of Schur functions (with S. Katz), *Duke Math. J.* 36:343–352 (1969).
- 103 Singular value inequalities, *J. London Math. Soc.* 44:118–120 (1969).
- 104 Symmetric positive definite multilinear functionals with a given automorphism (with S. Pierce), *Pacific J. Math.* 31:119–132 (1969).
- 105 Subpermanents, *Amer. Math. Monthly* 76:530–533 (1969).
- 106 Inequalities for some monotone matrix functions (with P. J. Nikolai), *Canad. J. Math.* 21:485–494 (1969).

- 107 Inequalities for matrix functions of combinatorial interest, *SIAM J. Appl. Math.* 17:1023–1031 (1969).
- 108 Spectral properties of higher derivations on symmetry classes of tensors, *Bull. Amer. Math. Soc.* 75:1303–1307 (1969).
- 109 Solution to advanced problem 63-2, *SIAM Rev.*, 1970.
- 110 A generalization of the unitary group (with W. R. Gordon), *Linear Algebra Appl.* 3:225–247 (1970).
- 111 Inequalities for submatrices, in *Inequalities—II* (proceedings of the Second Symposium on Inequalities, U.S. Air Force Academy, Colorado, 14–22 Aug. 1967), Academic, New York, 1970, pp. 223–240.
- 112 Some results on unitary matrix groups (with M. Newman), *Linear Algebra Appl.* 3:173–178 (1970).
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